MATH 1050 - EXAM 5 REVIEW KEY

Note Title 12/18/2013

1. $\frac{7!}{4!} = \frac{7\cdot6\cdot5\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 7\cdot6\cdot5 = \boxed{210}$

2. a) $\{s_n\} = \{4n-2\}$

$s_1 = 4(1)-2 = 2$

$s_2 = 4(2)-2 = 6$

$s_3 = 4(3)-2 = 10$

$s_4 = 4(4)-2 = 14$

$s_5 = 4(5)-2 = 18$

$\boxed{2, 6, 10, 14, 18, \ldots}$

b) $\{a_n\} = \{2n^2+n\}$

$a_1 = 2(1)^2+1 = 3$

$a_2 = 2(2)^2+2 = 10$

$a_3 = 2(3)^2+3 = 21$

$a_4 = 2(4)^2+4 = 36$

$a_5 = 2(5)^2+5 = 55$

$\boxed{3, 10, 21, 36, 55, \ldots}$

3. $a_1 = 8$

$a_n = 5a_{n-1} + 2$

$a_1 = 8$

$a_2 = 5a_1 + 2 = 5(8)+2 = 42$

$a_3 = 5a_2 + 2 = 5(42)+2 = 212$

$a_4 = 5a_3 + 2 = 5(212)+2 = 1062$

$\boxed{8, 42, 212, 1062 \ldots}$

4. $\sum_{k=1}^{n} (3k-1) = [3(1)-1] + [3(2)-1] + [3(3)-1] + \ldots + [3n-1]$

$= \boxed{2+5+8+\ldots+3n-1}$

5. $4^3 + 5^3 + 6^3 + \ldots + 13^3 = \boxed{\sum_{k=4}^{13} k^3}$

6. $\sum_{k=2}^{5} (3k+7) = [3(2)+7] + [3(3)+7] + [3(4)+7] + [3(5)+7]$

$= 13 + 16 + 19 + 22 = \boxed{70}$

7. $a_1 = 5 \quad d = 3$

$a_n = a_1 + (n-1)d$

$a_n = 5 + (n-1)3$

$a_n = 5 + 3n - 3$

$\boxed{a_n = 3n+2}$

$a_{18} = 3(18)+2 = \boxed{56}$

-or-

$a_{18} = 5 + (18-1)(3) = \boxed{56}$

8. 28 rows (↑ n) 18 seats in row 1 (↑ a_1) 18, 20, 22, ...

$d = 2$

$S_n = \frac{n}{2}(a_1 + a_n)$

$a_{28} = 18 + (28-1)(2) = 72$ seats on 28th row

$S_{28} = \frac{28}{2}(18 + 72) = \boxed{1260 \text{ seats}}$

9. $a_7 = -47$ $a_{13} = -101$

$a_n = a_1 + (n-1)d$

a_7: $-1(-47 = a_1 + 6d)$

a_{13}: $-101 = a_1 + 12d$

$47 = -a_1 - 6d$

$-54 = 6d$

$\boxed{d = -9}$

$-47 = a_1 + 6(-9)$

$-47 = a_1 - 54$

$\boxed{a_1 = 7}$

Recursive Formula:

$\boxed{a_1 = 7 \quad a_n = a_{n-1} - 9}$

10. $(-5) + (-2) + 1 + 4 + \ldots + 76$

$a_1 = -5$ $a_n = 76$ $d = 3$ $n = ?$

$76 = -5 + (n-1)(3)$

$76 = -5 + 3n - 3$

$76 = 3n - 8$

$84 = 3n$

$n = 28$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{28} = \frac{28}{2}(-5 + 76) = 14(71) = \boxed{994}$

11. 10th term of $-1, \frac{1}{2}, -\frac{1}{4}, \ldots$

$a_1 = -1$ $r = -\frac{1}{2}$

$a_n = a_1 r^{n-1}$

$a_{10} = (-1)(-\frac{1}{2})^{10-1} = (-1)(-\frac{1}{2})^9 = \boxed{\frac{1}{512}}$

12. $a = 3$ $r = 4$

$\boxed{a_n = 3(4)^{n-1}}$

$a_6 = 3(4)^{6-1} = 3(4)^5 = 3(1024) = \boxed{3072}$

13. $2, 6, 18, 54, 162, \ldots$

$a_1 = 2 \quad r = 3$

$\boxed{a_n = 2(3)^{n-1}}$

14. $\sum_{k=1}^{5} \left(\frac{1}{2}\right)(2)^k$

$a_1 = \left(\frac{1}{2}\right)(2)^1 = 1$

$r = 2$

$n = 5$

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

$S_5 = 1\left(\frac{1-2^5}{1-2}\right) = \boxed{31}$

-or- write it out

$\left(\frac{1}{2}\right)(2)^1 + \left(\frac{1}{2}\right)(2)^2 + \left(\frac{1}{2}\right)(2)^3 + \left(\frac{1}{2}\right)(2)^4 + \left(\frac{1}{2}\right)(2)^5$

$= 1 + 2 + 4 + 8 + 16$

$= \boxed{31}$

15. a) $\sum_{k=1}^{\infty} \frac{2}{3} \cdot 2^{k-1}$

$r = 2$

$\boxed{\text{diverges}}$

b) $4 - 2 + 1 - \frac{1}{2} + \ldots$

$a_1 = 4 \quad r = -\frac{1}{2}$

$\boxed{\text{converges}}$

$S = \frac{a_1}{1-r} = \frac{4}{1-\left(-\frac{1}{2}\right)} = \frac{4}{3/2} = 4\left(\frac{2}{3}\right) = \boxed{\frac{8}{3}}$

16. a) $\{-5n + 2\}$

linear

$\boxed{\begin{array}{c}\text{arithmetic} \\ d = -5\end{array}}$

b) $\{4n^2 + 7\}$

quadratic

$\boxed{\text{neither}}$

c) $\{3^{2n}\} = \{9^n\}$

exponential

$\boxed{\begin{array}{c}\text{geometric} \\ r = 9\end{array}}$

17. a) $\binom{10}{4} = \frac{10!}{4!\,6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{210}$

b) $\binom{6}{5} = \frac{6!}{5!\,1!} = \frac{6}{1} = \boxed{6}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

18. a) $(x-3)^5$ Use 1 5 10 10 5 1

$= 1(x)^5 + 5(x)^4(-3) + 10(x)^3(-3)^2 + 10(x)^2(-3)^3 + 5(x)(-3)^4 + 1(-3)^5$

$= \boxed{x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243}$

b) $(2x+5)^4$ Use 1 4 6 4 1

$= 1(2x)^4 + 4(2x)^3(5) + 6(2x)^2(5)^2 + 4(2x)(5)^3 + 1(5)^4$

$= \boxed{16x^4 + 160x^3 + 600x^2 + 1000x + 625}$

c) $(4x - y^2)^6$ Use 1 6 15 20 15 6 1

$= 1(4x)^6 + 6(4x)^5(y^2) + 15(4x)^4(y^2)^2 + 20(4x)^3(y^2)^3 + 15(4x)^2(y^2)^4 + 6(4x)(y^2)^5 + 1(y^2)^6$

$= \boxed{4096x^6 - 6144x^5y^2 + 3840x^4y^4 - 1280x^3y^6 + 240x^2y^8 - 24xy^{10} + y^{12}}$

19. a) Coefficient of x^2 in $(3x+2)^6$

$\binom{6}{4}(3x)^2(2)^4$

$= 15(9x^2)(16)$ $\boxed{2160}$

$= 2160x^2$

b) 4th term in $(5x-1)^4$ x^4 x^3 x^2 x

$= \binom{4}{3}(5x)(-1)^3$

$= 4(5x)(-1)$

$= \boxed{-20x}$